VECTOR (OR CROSS) PRODUCT OF TWO VECTORS

Properties of vector (or cross) product

Angle between two vectors

Lagrange's identity

if $\vec{a}, \vec{b}$ are any two vectors then $\left(\vec{a} \times \vec{b}\right)^2 = |\vec{a}|^2 \left|\vec{b}\right|^2 - \left(\vec{a}.\vec{b}\right)^2$

Area of a parallelogram

Area of a triangle

Question 1.

(i) If $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} + 5\hat{j} - 2\hat{k}$, then find $|\vec{a} \times \vec{b}|$. (NCERT)

(ii) Find the magnitude of $\vec{a}$, where $\vec{a} = (\hat{i} + 3\hat{j} - 2\hat{k}) \times (-\hat{i} + 3\hat{k})$.

(iii) If $\vec{a} = 3\hat{i} - 5\hat{j}$, $\vec{b} = 6\hat{i} + 3\hat{j}$ are two vectors and $\vec{c}$ is a vector such that $\vec{c} = \vec{a} \times \vec{b}$, then find $|\vec{a}| : |\vec{b}| : |\vec{c}|$.

Question 2.

(i) Find the angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes 1 and 2 respectively when $|\vec{a} \times \vec{b}| = \sqrt{3}$.

(ii) If vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = 3, |\vec{b}| = 2/3$ and $\vec{a} \times \vec{b}$ is a unit vector then find the angle between $\vec{a}$ and $\vec{b}$.

(iii) If $|\vec{a}| = 2$, $|\vec{b}| = 7$ and $\vec{a} \times \vec{b} = 3\hat{i} + 2\hat{j} + 6\hat{k}$, find the angle between $\vec{a}$ and $\vec{b}$.

Question 3.

Find the angle between two vectors $\vec{a}$ and $\vec{b}$ if $|\vec{a} \times \vec{b}| = \vec{a} \cdot \vec{b}$. (NCERT)

Question 4.

(i) Find $\vec{a} \cdot \vec{b}$ if $|\vec{a}| = 2$, $|\vec{b}| = 5$ and $|\vec{a} \times \vec{b}| = 8$.

(ii) If $|\vec{a}| = 4$; $|\vec{b}| = 2$ and angle between $|\vec{a}|$ and $|\vec{b}|$ is $\frac{\pi}{3}$, find $(\vec{a} \times \vec{b})^2$.

(iii) If vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = 1/2$, $|\vec{b}| = 4/\sqrt{3}$ and $|\vec{a} \times \vec{b}| = 1/\sqrt{3}$ then find $|\vec{a} \cdot \vec{b}|$.

Question 5.

(i) Find the value of λ if $(2\hat{i}+6\hat{j}+14\hat{k})\times(\hat{i}-\lambda\hat{j}+7\hat{k})=\vec{0}$.

(ii) Find the value of p if $(2\hat{i}+6\hat{j}+27\hat{k})\times(\hat{i}+3\hat{j}+p\hat{k})=\vec{0}$.

(iii) Find λ and μ $(2\hat{i}+6\hat{j}+27\hat{k})\times(\hat{i}+\lambda\hat{j}+\mu\hat{k})=\vec{0}$. (NCERT)

$$(2\hat{i}+6\hat{j}+14\hat{k})\times(\hat{i}-\lambda\hat{j}+7\hat{k})=\vec{0}$$

$$\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 6 & 14\\ 1 & -\lambda & 7\end{vmatrix}=\vec{0}$$

$$\vec{0}=0\hat{i}+0\hat{j}+0\hat{k}$$

$$\hat{i}(42+14\lambda)-\hat{j}(14-14)+\hat{k}(-2\lambda-6)=0\hat{i}+0\hat{j}+0\hat{k}$$

$$42+14\lambda=0$$

$$14\lambda=-42$$

$$\lambda=-42/14=-3$$

$$-2\lambda-6=0$$

$$\lambda=\frac{-6}{2}=-3$$

$(2\hat{i}+6\hat{j}+27\hat{k}) \times (\hat{i}+3\hat{j}+p\hat{k}) = \vec{0}$ $\quad p = ?$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 6 & 27 \\ 1 & 3 & p \end{vmatrix} = \vec{0}$$

$$\hat{i}(6p-81) - \hat{j}(2p-27) + \hat{k}(6-6) = \vec{0}$$

$$\hat{i}(6p-81) - \hat{j}(2p-27) + 0\hat{k} = 0\hat{i}+0\hat{j}+0\hat{k}$$

$6p-81=0$ $\qquad$ $2p-27=0$

$6p=81$ $\qquad$ $2p=27$

$\boxed{p=\frac{81}{6}=\frac{27}{2}}$ $\qquad$ $\boxed{p=\frac{27}{2}}$

$$(2\hat{i}+6\hat{j}+27\hat{k}) \times (\hat{i}+\lambda\hat{j}+\mu\hat{k}) = \vec{0}$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 6 & 27 \\ 1 & \lambda & \mu \end{vmatrix} = 0\hat{i}+0\hat{j}+0\hat{k}$$

$$\hat{i}(6\mu-27\lambda)-\hat{j}(2\mu-27)+\hat{k}(2\lambda-6) = 0\hat{i}+0\hat{j}+0\hat{k}$$

$6\mu-27\lambda=0$

$2\mu-27=0$

$2\mu=27$

$\boxed{\mu=\frac{27}{2}}$

$2\lambda-6=0$

$2\lambda=6$

$\boxed{\lambda=3}$

Question 6.

(i) Find a unit vector perpendicular to the two vectors $\hat{i} + 2\hat{j} - \hat{k}$ and $2\hat{i} + 3\hat{j} + \hat{k}$. (ISC 2004)

(ii) Find a unit vector perpendicular to $\vec{a}$ and $\vec{b}$, where $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$.

$$\vec{a} \times \vec{b} = |\vec{a}|\,|\vec{b}| \sin\theta\, \hat{n}$$

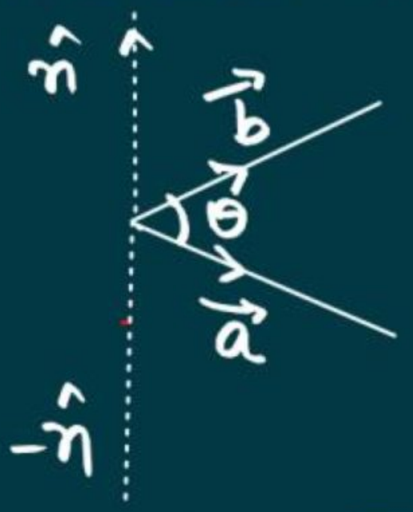
$$|\hat{n}| = 1$$

Let $\vec{a} = \hat{i} + 2\hat{j} - \hat{k}$ $\quad \vec{b} = 2\hat{i} + 3\hat{j} + \hat{k}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 2 & 3 & 1 \end{vmatrix}$$

$$= \hat{i}(2+3) - \hat{j}(1+2) + \hat{k}(3-4)$$

$$\vec{a} \times \vec{b} = 5\hat{i} - 3\hat{j} - \hat{k}$$



$$|\vec{a} \times \vec{b}| = \sqrt{(5)^2 + (-3)^2 + (-1)^2}$$

$$= \sqrt{25 + 9 + 1}$$

$$= \sqrt{35}$$

$$\hat{n} = \pm \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|} = \frac{5\hat{i} - 3\hat{j} - \hat{k}}{\sqrt{35}}$$ Ans

Question 7.

(i) What conclusion can you draw about vectors $\vec{a}$ and $\vec{b}$ when $\vec{a} \times \vec{b} = \vec{0}$ and $\vec{a} \cdot \vec{b} = \vec{0}$? (NCERT)

(ii) If either $\vec{a} = 0$ or $\vec{b} = 0$, then $\vec{a} \times \vec{b} = \vec{0}$. Is the converse true? Justify your answer with an example. (NCERT)

(i) $\vec{a} \times \vec{b} = 0$, $\vec{a} = 0$ OR $\vec{b} = 0$ $\quad \vec{a} \parallel \vec{b}$ (Collinear)

$\vec{a} \cdot \vec{b} = 0$, $\vec{a} = 0$ OR $\vec{b} = 0$ $\quad \vec{a} \perp \vec{b}$ $\quad \theta = \frac{\pi}{2}$

(ii) $\vec{a} = 0$, $\vec{b} = 0$ $\quad \vec{a} \times \vec{b} = 0$ $\quad$ True

$\vec{a} \times \vec{b} = 0$ $\quad$ False.

Question 8.

Find λ such that $\vec{a} = \hat{i} + \lambda\hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} + 2\hat{j} + 9\hat{k}$ are parallel.

Hint: $\vec{a} \times \vec{b} = \vec{0}$

$$\vec{a} = \hat{i} + \lambda\hat{j} + 3\hat{k} \qquad \vec{b} = 3\hat{i} + 2\hat{j} + 9\hat{k}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & \lambda & 3 \\ 3 & 2 & 9 \end{vmatrix}$$

$$0 = \hat{i}(9\lambda - 6) - \hat{j}(9 - 9) + \hat{k}(2 - 3\lambda)$$

$$\hat{i}(9\lambda - 6) - 0 + \hat{k}(2 - 3\lambda) = 0$$

$$9\lambda - 6 = 0$$

$$\lambda = 6/9 = 2/3$$

Question 9.

(i) $(\hat{i} \times \hat{j}).\hat{k} + \hat{i}.\hat{j}$

(ii) $(\hat{j} \times \hat{k}).\hat{i} + (\hat{i} \times \hat{k}).\hat{j} + (\hat{i} \times \hat{j}).\hat{k}$

(i) $(\hat{i} \times \hat{j}) \cdot \hat{k} + \hat{i} \cdot \hat{j}$

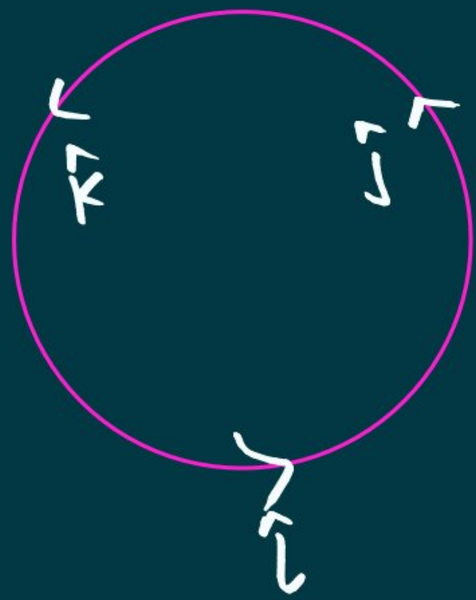
$\hat{k} \cdot \hat{k} + 0$

$1 + 0 = 1$

(ii) $(\hat{j} \times \hat{k}) \cdot \hat{i} + (\hat{i} \times \hat{k}) \cdot \hat{j} + (\hat{i} \times \hat{j}) \cdot \hat{k}$

$\hat{i} \cdot \hat{i} + (-\hat{j}) \cdot \hat{j} + \hat{k} \cdot \hat{k}$

$1 - 1 + 1 = 1$



Imp.

$\hat{i} \times \hat{j} = \hat{k}$

$\hat{j} \times \hat{k} = \hat{i}$

$\hat{k} \times \hat{i} = \hat{j}$

$\hat{i} \times \hat{i} = 0$

$\hat{j} \times \hat{j} = 0$

$\hat{k} \times \hat{k} = 0$

$\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$

$\cos 0 = 1$

$\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$

Question 10.

Find the area of the parallelogram whose adjacent sides are represented by the vectors

(i) $3\hat{i}+\hat{j}+4\hat{k}$ and $\hat{i}-\hat{j}+\hat{k}$ (NCERT)

(ii) $2\hat{i}-3\hat{k}$ and $4\hat{j}+2\hat{k}$

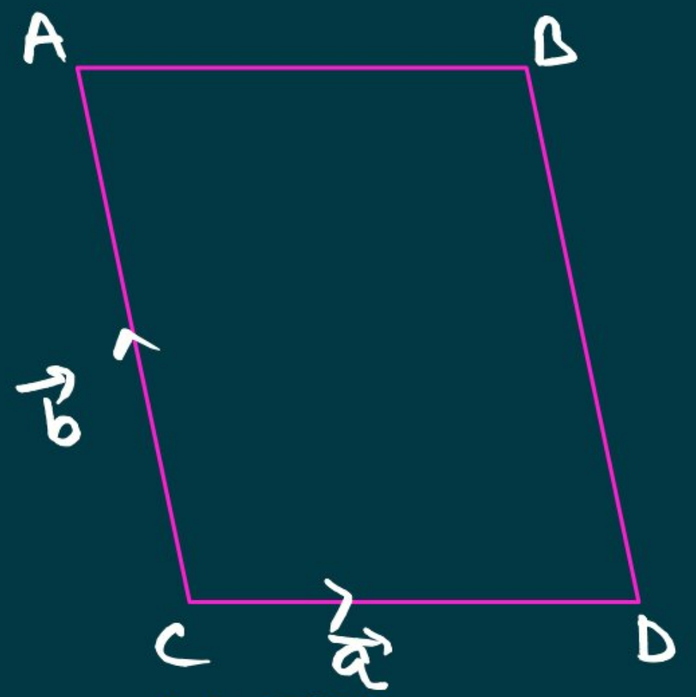
(i) $\vec{a}=3\hat{i}+\hat{j}+4\hat{k}$ $\quad \vec{b}=\hat{i}-\hat{j}+\hat{k}$

Area of a parallelogram $= |\vec{a}\times\vec{b}|$

$$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 1 & 4\\ 1 & -1 & 1\end{vmatrix}$$

$$=\hat{i}(1+4)-\hat{j}(3-4)+\hat{k}(-3-1)$$

$$\vec{a}\times\vec{b}=5\hat{i}+\hat{j}-4\hat{k}$$



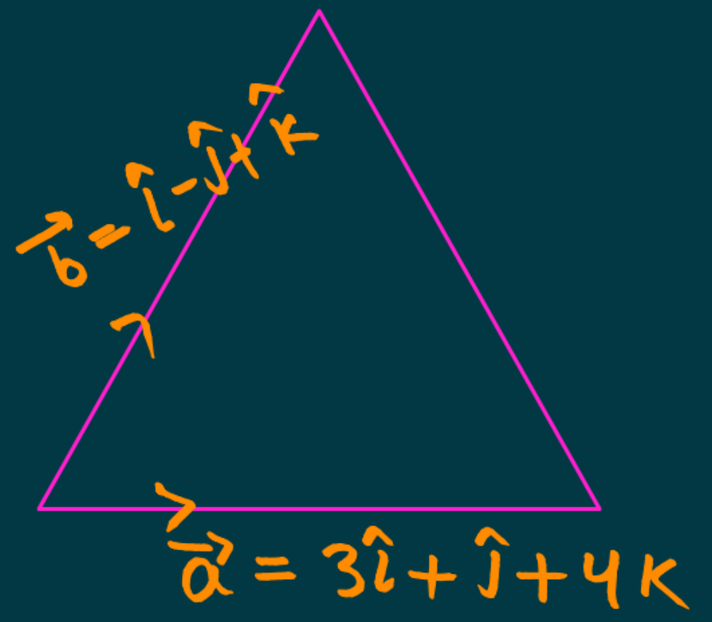
Area $= |\vec{a}\times\vec{b}|$

$$=\sqrt{(5)^2+1+(-4)^2}$$

$$=\sqrt{25+1+16}=$$ Sq. Unit

⑪ $\vec{a}=3\hat{i}+\hat{j}+4\hat{k}$ and $\vec{b}=\hat{i}-\hat{j}+\hat{k}$

Area of a triangle $=\frac{1}{2}|\vec{a}\times\vec{b}|$



$$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 1 & 4\\ 1 & -1 & 1\end{vmatrix}$$

$$=\hat{i}(1+4)-\hat{j}(3-4)+\hat{k}(-3-1)$$

$$\vec{a}\times\vec{b}=5\hat{i}-\hat{j}-4\hat{k}$$

$$|\vec{a}\times\vec{b}|=\sqrt{25+1+16}=\sqrt{42}$$

$$\begin{array}{r}17\\ 25\\ \hline 42\end{array}$$

Area $=\frac{1}{2}\times\sqrt{42}$ Sq. Unit.

(12) If $\vec{a} = 4\hat{i} + 3\hat{j} + 2\hat{k}$ and $\vec{b} = 3\hat{i} + 2\hat{k}$, then find $|\vec{b} \times 2\vec{a}|$

$$2\vec{a} = 2(4\hat{i} + 3\hat{j} + 2\hat{k})$$

$$= 8\hat{i} + 6\hat{j} + 4\hat{k}$$

$$\vec{b} \times 2\vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 0 & 2 \\ 8 & 6 & 4 \end{vmatrix}$$

$$= \hat{i}(0-12) - \hat{j}(12-16) + \hat{k}(18-0)$$

$$\vec{b} \times 2\vec{a} = -12\hat{i} - 4\hat{j} + 18\hat{k}$$

$$|\vec{b} \times 2\vec{a}| = \sqrt{(-12)^2 + (-4)^2 + (18)^2}$$

(13)(i) $\vec{a} = 3\hat{i} - \hat{j} + 2\hat{k}$ $\vec{b} = 2\hat{i} + 3\hat{j} + \hat{k}$ then find $(\vec{a} + 2\vec{b}) \times (2\vec{a} - \vec{b})$

$$\vec{a} + 2\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k} + 4\hat{i} + 6\hat{j} + 2\hat{k}$$

$$\vec{a} + 2\vec{b} = 7\hat{i} + 5\hat{j} + 4\hat{k}$$

$$2\vec{a} - \vec{b} = 6\hat{i} - 2\hat{j} + 4\hat{k} - 2\hat{i} - 3\hat{j} - \hat{k}$$

$$2\vec{a} - \vec{b} = 4\hat{i} - 5\hat{j} + 3\hat{k}$$

$$(\vec{a} + 2\vec{b}) \times (2\vec{a} - \vec{b}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & 5 & 4 \\ 4 & -5 & 3 \end{vmatrix}$$

$$= \hat{i}(15 + 20) - \hat{j}(21 - 16) + \hat{k}(-35 - 20)$$

$$= 35\hat{i} - 7\hat{j} - 55\hat{k}$$